GOLDEN GATE TO IIT

SOLUTION TO SAMPLE ENTRANCE TEST

1. The given equation is $\frac{xy+1}{y-x}=3$.

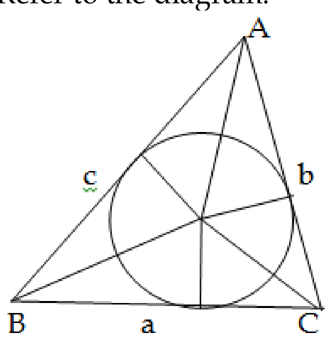
Rewriting this equation we get $y(x-3)=-3x-1$ which gives $y=\frac{1+3x}{3-x}$. Since x and y are natural numbers, we have $x>0, y>0$. Hence $3x+1>0$. Hence also $3-x>0$ and since x is natural, we must have $x=1$ or $x=2$. Hence for $x=1$, $y=2$ and for $x=2, y=7$. Hence the answer pairs are $(1,2)$ and $(2,7)$.

2. The possible implications are :
   1. B divides segment AC in some ratio.
   2. Area of ΔABC is 0.
   3. $l(AB)+l(BC)=l(AC)$
   4. A, B and C can not lie on the same circle.

3. $a^x=ab \Rightarrow a^{x-1}=b$ ….(1) and also we have $b^y=ab \Rightarrow b^{y-1}=a$ $\Rightarrow b=a^{1/(y-1)}$ ..(2) Now from (1) and (2) we get $a^{x-1}=a^{1/(y-1)}$. This means $x-1=\frac{1}{y-1} \Rightarrow (x-1)(y-1)=1$

Hence $xy-x-y+1=1 \Rightarrow xy=x+y$

4. Refer to the diagram.



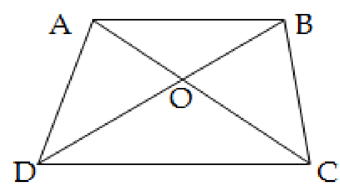
Let the centre of the circle be I. Let the radius of the incircle be *r*. Then the area of the triangle ABC is given by $\Delta=\Delta AIB+\Delta BIC+\Delta CIA$.

$\Delta=\frac{1}{2}rc+\frac{1}{2}ra+\frac{1}{2}rb$

$=\frac{1}{2}r(a+b+c)$ as required.

5. We have to prove that x can not be rational. Let us assume the contrary. i.e. let x be rational. Then by definition of a rational number there exist natural numbers p and q such that $x=\frac{p}{q}$. Hence the given equation becomes $2^{p/q}=7$. Hence we get $2^p=7^q$. Now for a natural p, 2^p is even while for a natural q, 7^q is odd. Hence we get the contradiction that an even no. = an odd number. Hence x can not be rational.

6. Consider the following diagram.



We are given that $\frac{AB}{DC}=\frac{1}{2}$. Also $\angle ABO=\angle CDO$ and $\angle BAO=\angle DCO$. Hence $\Delta AOB \sim \Delta COD$. Hence we have $\frac{AB}{CD}=\frac{AO}{CO}=\frac{BO}{DO}=\frac{1}{2}$. This proves the result. i.e. O is the point of trisection of both the diagonals.

7. Given $\sqrt{2}\left(\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}\right)$

$=\sqrt{4+2\sqrt{3}}-\sqrt{4-2\sqrt{3}}$

$=\sqrt{\left(\sqrt{3}+1\right)^2}-\sqrt{\left(\sqrt{3}-1\right)^2}$

$=\left(\sqrt{3}+1\right)-\left(\sqrt{3}-1\right)=2$ which is even.

8. Consider the following diagram

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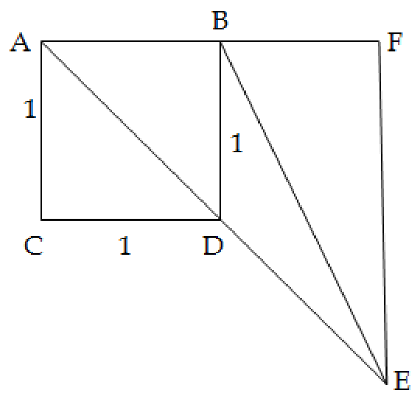


Extend AB to F such that $BF \perp EF$. Then by midpoint theorem in ΔAEF we get $BF = 1, EF = 2$. Hence in right angled triangle BEF we get

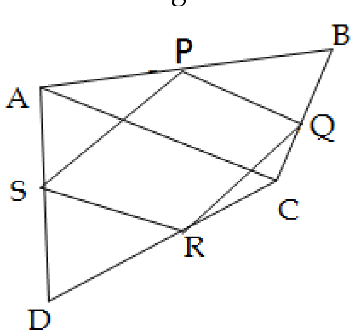
$l(BE) = \sqrt{(l(BF))^2 + (l(EF))^2} = \sqrt{1+4}$

$= \sqrt{5}$

9. The given equation is $3^{2x} - 3^{x+1} - 3^{x-1} + 1 = 0$ which can be written as $(3^x)^2 - 3.3^x - \frac{3^x}{3} + 1 = 0$.

Now put $3^x = y$. Then the equation becomes $y^2 - 3y - \frac{y}{3} + 1 = 0$ which gives $3y^2 - 10y + 3 = 0$ which is $(3y-1)(y-3) = 0 \Rightarrow y = 3, \frac{1}{3}$ which gives $3^x = 3$ or $3^x = 3^{-1}$ which gives $x = \pm 1$

10. Refer to the diagram.



Join AC. In ΔABC, by midpoint theorem, $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$.

Similarly in ΔADC, $SR \parallel AC$ and $SR = \frac{1}{2}AC$. Hence we get $SR = PQ$ and $SR \parallel PQ$. Hence quadrilateral PQRS is a parallelogram.

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